


DataSig
A rough path between
mathematics and data science



Mathematical
Institute

16th Oxford-Berlin Young Researchers' Meeting on Applied Stochastic Analysis

8-10 December 2022



Oxford
Mathematics



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8 **Participants**



1. Welcome

It is our great pleasure to welcome you to the 16th Oxford-Berlin Young Researchers Meeting on Applied Stochastic Analysis. We hope you enjoy a productive meeting!

Scientific Board

Terry Lyons (University of Oxford)
Peter Friz (TU and WIAS Berlin)

Conference organisers

Martin Geller (University of Oxford)
Jason Rader (University of Oxford)
Philipp Forstner (TU Berlin)
Toyomu Matsuda (FU Berlin)

Presentations

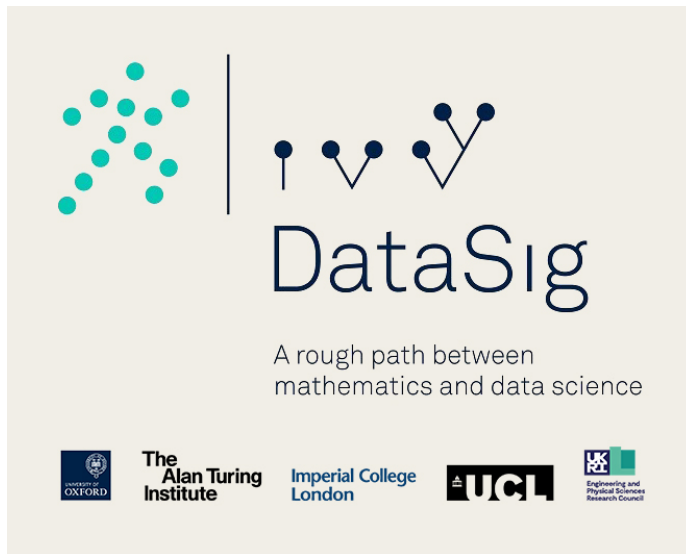
All talks will be held in the Mathematical Institute. On 8 December, talks will be in Lecture Room L2, and on 9 and 10 December, they will be in Lecture Room L3. Talks will be 15 minutes in length, with 5 minutes for questions after each talk. There will also be coffee breaks for discussion.

Formal dinner

This will take place on 8 December at 7.30pm. If you have not booked already, we may still be able to accommodate you. Please email rader@maths.ox.ac.uk, copying in geller@maths.ox.ac.uk, if you would like to attend but have not registered.

The address of the main Mathematical Institute building, known as the Andrew Wiles Building, is “Andrew Wiles Building, Woodstock Rd, Oxford OX2 6GG”. Once you have arrived at the Oxford railway station, it is an approx. 20-minute walk, but putting the above address on Google Maps will give you options in terms of buses that may take you closer.

Supporting institution



This meeting is generously supported by the DataSig programme (EPSRC EP/S026347/1).



2. Schedule

Thursday, 8 December

9.30–10am	Welcome		
10–10.05am	Martin Geller (University of Oxford)	<i>Introduction</i>	10
10.05–10.25am	Hannes Kern (Technische Universität Berlin)	<i>A bi-algebra map for branched rough differential equations on manifolds</i>	10
10.25–10.45am	Emilio Ferrucci (University of Oxford)	<i>On the Wiener Chaos Expansion of the Signature of a Gaussian Process</i>	10
10.45–11am	Coffee Break		
11–11.20am	Nicola Muça Cirone (Imperial College London)	<i>Signatures and the infinite-width-depth limit of Data Driven ResNets</i>	10
11.20–11.40am	Toyomu Matsuda (FU Berlin)	<i>Regularization by noise for $L^q L^p_x$-drift</i>	11
11.40am–12pm	Rosa Preiß (TU Berlin)	<i>Smooth Rough Paths</i>	11
12–1.30pm	Lunch Break		
1.30–1.50pm	Henri Elad Altman (FU Berlin)	<i>Local times and global limits.</i>	11
1.50–2.10pm	Leonard Schmitz (Greifswald University)	<i>Two-parameter sums signatures and corresponding quasisymmetric functions</i>	11
2.10–2.30pm	Florian Bechtold (Bielefeld University)	<i>Non-linear Young equations in the plane and pathwise regularization by noise for the stochastic wave equation</i>	11
2.30–3pm	Coffee Break		
3–3.20pm	Guido Gazzani (Universität Wien)	<i>Joint calibration of SPX and VIX options with signature-based models</i>	12
3.20–3.40pm	Lingyi Yang (University of Oxford)	<i>Economic nowcasting with signatures</i>	12

3.40–4pm	Milena Vuletić (University of Oxford)	<i>Simulating Arbitrage-Free Implied Volatility Surfaces</i>	13
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Friday, 9 December

9.30–9.50am	Benjamin Walker (University of Oxford)	<i>Neural Controlled Differential Equations: The Log-ODE Method</i>	16
9.50–10.10am	Simon Breneis (Weierstrass Institute)	<i>An error representation formula for the Log-ODE method</i>	14
10.10–10.30am	Darrick Lee (University of Oxford)	<i>Capturing Graphs with Hypo-Elliptic Diffusion</i>	14
10.30–11am	Coffee Break		
11–11.20am	Satoshi Hayakawa (University of Oxford)	<i>Positively Weighted Kernel Quadrature via Subsampling</i>	14
11.20–11.40am	Thomas Wagenhofer (TU Wien)	<i>Weak error estimates for rough volatility models</i>	14
11.40am–12pm	Luca Pelizzari (Weierstrass Institute)	<i>Polynomial Volterra processes</i>	15
12–1.30pm	Lunch Break		
1.30–1.50pm	Henry Chiu (Imperial College London)	<i>Mathematical finance without probability</i>	12
1.50–2.10pm	Khoa Lê (University of Leeds)	<i>Quantitative John–Nirenberg inequality for stochastic processes of bounded mean oscillation</i>	16
2.10–2.30pm	Sarah-Jean Meyer (University of Oxford)	<i>A Forward-Backward SDE Approach to the sine-Gordon QFT</i>	16
2.30–3pm	Coffee Break		
3–3.20pm	Weile Weng (TU Berlin)	<i>Quenched functional CLT for random walks in degenerate doubly stochastic random environments</i>	17
3.20–3.40pm	Sebastian Ertel (TU Berlin)	<i>Analysis of the Ensemble Kalman–Bucy Filter for correlated observation noise</i>	17
3.40–4pm	Jacob Armstrong Goodall (University of Edinburgh)	<i>Dobrushin and Steif Metrics are Equal</i>	17

Saturday, 10 December

9.30–9.50am	Ioannis Gasteratos (Imperial College London)	<i>Large deviations of slow-fast systems driven by fractional Brownian motion</i>	17
9.50–10.10am	Adrian Martini (University of Oxford)	<i>SPDE Approximations to Singular Mean-Field Systems</i>	18
10.10–10.30am	Harprit Singh (Imperial College London)	<i>Singular Equations on homogeneous Lie Groups</i>	18
10.30–11am	Coffee Break		

11–11.20am	Rhys Steele (Imperial College London)	<i>The BPHZ Theorem for Models via Spectral Gap</i>	18
11.20–11.40am	Lukas Gräfner (FU Berlin)	<i>Unique $L^2(\mu)$-semigroups for some (sub-) critical singular SPDEs</i>	19
11.40am–12pm	Jean-David Jacques (Sorbonne Université)	<i>Novikov algebra structure for multi-indices, aromatic B-series and numerical methods.</i>	19



3. Theory of Rough Paths

3.1 A bi-algebra map for branched rough differential equations on manifolds

Hannes Kern, Technische Universität Berlin

In a paper published in 2020, C. Curry et al. presented a way to solve rough differential equations on homogeneous spaces by mapping signatures into differential operators. The construction of this space heavily relied on the post-Lie algebra structure of the underlying vector fields.

Two years later, J. Armstrong et al. published a paper on branched rough paths on manifolds, solving rough differential equations on manifolds with a connection, even if the vector fields do not form a post-Lie algebra. In this talk, we connect the two articles by presenting a bi-algebra map, which generalizes the post-Lie algebra map described by Curry et al. to the setting of Armstrong et al.

3.2 On the Wiener Chaos Expansion of the Signature of a Gaussian Process

Emilio Ferrucci, University of Oxford

We compute the Wiener chaos decomposition of the signature for a class of Gaussian processes, which contains fractional Brownian motion (fBm) with Hurst parameter H in $(1/4, 1)$. At level 0, our result yields an expression for the expected signature of such processes, which determines their law [Chevyrev Lyons 2016]. In particular, this formula simultaneously extends both the one for $1/2 < H$ -fBm [Baudoin Coutin 2007] and the one for Brownian motion ($H = 1/2$) [Fawcett 2003], to the general case $H > 1/4$, thereby resolving an established open problem. Other processes studied include continuous and centred Gaussian semimartingales. This is joint work with Thomas Cass: <https://arxiv.org/abs/2207.08422>

3.3 Signatures and the infinite-width-depth limit of Data Driven ResNets

Nicola Muca Cirone, University of Oxford

We consider recurrent versions of ResNets, randomly initialised with Gaussian weights and biases, and we identify them as Reservoirs. Then, we show that in the infinite-width-depth limit, these architectures converge to Gaussian Processes with covariances satisfying certain differential-integral equations, varying according to the non-linear activation function choice. In the simplified setting where the chosen activation function is the identity, the limiting covariance agrees with the signature kernel and satisfies a classical Goursat PDE. The Reproducing Kernel Hilbert Spaces of this new family of kernels form rich classes of functions on path space that we conjecture can justify rigorously the expressiveness via universal approximation of Randomized Signatures. We end by corroborating our results and conjectures with experimental data and discuss avenues for future work.

3.4 Regularization by noise for $L_t^q L_x^p$ -drift

Toyomu Matsuda, FU Berlin

I will report a progress on a joint work with Oleg Butkovsky (WIAS Berlin) and Khoa Lê (Leeds), where we study a stochastic differential equation with $L_t^q L_x^p$ -drift and additive fractional noise with Hurst parameter less than $\frac{1}{2}$. We discuss the uniqueness of strong solutions and the stability with respect to the drift and the initial condition. It turns out that these problems are a nice playground for recent techniques of rough analysis, namely stochastic sewing and VMO processes.

3.5 Smooth Rough Paths

Rosa Preiß, TU Berlin

Joined work with Carlo Bellingeri, Peter Friz and Sylvie Paycha.

We introduce the class of “smooth rough paths” and study their main properties. Working in a smooth setting allows us to discard sewing arguments and focus on algebraic and geometric aspects. Specifically, we discuss a purely algebraic form of Lyons’ extension theorem, formulate “smooth rough differential equations” as standard ODEs and introduce a canonical sum of smooth rough paths.

3.6 Local times and global limits.

Henri Elad Altman, FU Berlin

We present scaling limit results for additive functionals of fractional Brownian motions, where the limits are given by some homogeneous distributions of the local times of the process. Our techniques rely on a local description of our processes provided by a singular version of the stochastic sewing lemma. This is joint work with Khoa Lê (University of Leeds).

3.7 Two-parameter sums signatures and corresponding quasisymmetric functions

Leonard Schmitz, Greifswald University

Quasisymmetric functions have recently been used in time series analysis as polynomial features that are invariant under, so-called, dynamic time warping. We extend this notion to data indexed by two parameters and thus provide warping invariants for images. We show that two-parameter quasisymmetric functions are complete in a certain sense, and provide a two-parameter quasi-shuffle identity. A compatible coproduct is based on diagonal concatenation of the input data, leading to a (weak) form of Chen’s identity.

3.8 Non-linear Young equations in the plane and pathwise regularization by noise for the stochastic wave equation

Florian Bechtold, Bielefeld University

We study pathwise regularization by noise for equations in the plane in the spirit of Catellier and Gubinelli. To this end we extend the notion of non-linear Young equations to a two dimensional domain and prove existence and uniqueness of such equations. We then use this concept in order to establish regularization by noise for stochastic differential equations in the plane by providing a space-time regularity estimate for the local time associated with the fractional Brownian sheet. As a further illustration of our results we also prove regularization of 1D wave equations with distributional non-linearity through a noisy boundary. Based on joint work with Fabian Harang (BI Norwegian Business School) and Nimit Rana (Imperial): <https://arxiv.org/abs/2206.05360>



4. Mathematical Finance

4.1 Joint calibration of SPX and VIX options with signature-based models

Guido Gazzani, Universität Wien

We consider asset price models whose volatility processes are described by linear functions of the (time extended) signature of a primary underlying process, which can range from a polynomial process to a general continuous semimartingale. One of our main focus lies on calibration, where we consider the problem of joint calibrating to both options written on the SP 500 and to options written on its volatility index, the VIX. Within our setting we provide both a closed-form expression for VIX futures and a tractable representation of the price process, which yield fast pricing of both SPX and VIX options. These act as a building block for our first promising results of the joint calibration.

4.2 Economic nowcasting with signatures

Lingyi Yang, University of Oxford

Economic nowcasting refers to the inference (“forecast”) of the current (“now”) state of the economy. This is necessary as key economic variables are often published with significant delays. The nowcasting literature focuses on the need to have fast, reliable estimates of these delayed indicators from available data sources. By embedding the observed data in continuous time, we can naturally handle missing data from mixed frequency and/or irregular sampling – issues often encountered when merging multiple data sources. We look at the nowcasting problem by applying regression on signatures and show that this simple linear model subsumes the popular Kalman filter in theory and performs well in practice.

4.3 Mathematical finance without probability

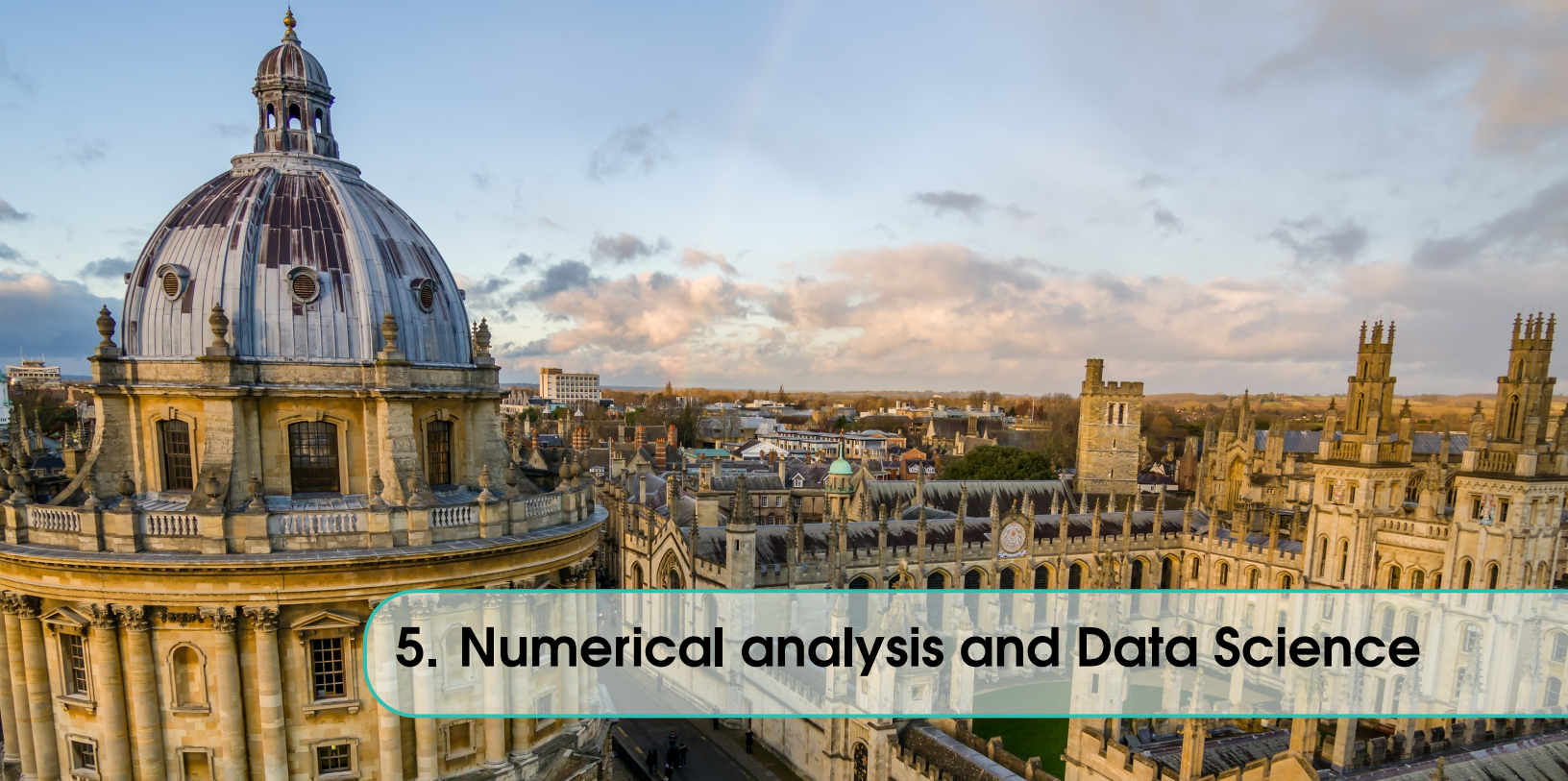
Henry Chiu, Imperial College London

We present a non-probabilistic, pathwise approach to continuous-time finance based on causal functional calculus. We introduce a definition of self-financing, free from any integration concept and show that the value of a self-financing portfolio is a pathwise integral (every self-financing strategy is a gradient) and that generic domain of functional calculus is inherently arbitrage-free. We then consider the problem of hedging a path-dependent payoff across a generic set of scenarios. We apply the transition principle of Isaacs in differential games and obtain a verification theorem for the optimal solution, which is characterised by a fully non-linear path-dependent equation. For the Asian option, we obtain explicit solution.

4.4 Simulating Arbitrage-Free Implied Volatility Surfaces

Milena Vuletić, University of Oxford

We present a computationally tractable method for simulating arbitrage free implied volatility surfaces. Our approach conciliates static arbitrage constraints with a realistic representation of statistical properties of implied volatility co-movements. We first illustrate how our method may be combined with a factor model for the implied volatility surface to generate dynamic scenarios for arbitrage-free implied volatility surfaces. We then introduce Volgan, a nonparametric generative model for implied volatility surfaces.



5. Numerical analysis and Data Science

5.1 Positively Weighted Kernel Quadrature via Subsampling

Satoshi Hayakawa, University of Oxford

We study kernel quadrature rules with convex weights. Our approach combines the spectral properties of the kernel with recombination results about point measures. This results in effective algorithms that construct convex quadrature rules using only access to i.i.d. samples from the underlying measure and evaluation of the kernel and that result in a small worst-case error. In addition to our theoretical results and the benefits resulting from convex weights, our experiments indicate that this construction can compete with the optimal bounds in well-known examples.

5.2 An error representation formula for the Log-ODE method

Simon Breneis, Weierstrass Institute

The Log-ODE method is a numerical method for solving rough differential equations. After explaining the idea behind the Log-ODE method, we show a new error representation formula for the discretization error of this method. In this formula, the global discretization error is expressed as a weighted sum of the local errors. Finally, we show some numerical examples, and construct an adaptive step size algorithm using this error representation formula.

5.3 Capturing Graphs with Hypo-Elliptic Diffusion

Darrick Lee, University of Oxford

Convolutional layers within graph neural networks operate by aggregating information about local neighbourhood structures; one common way to encode such substructures is through random walks. The distribution of these random walks evolves according to a diffusion equation defined using the graph Laplacian. We extend this approach by leveraging classic mathematical results about hypo-elliptic diffusions. This results in a novel tensor-valued graph operator, which we call the hypo-elliptic graph Laplacian. We provide theoretical guarantees and efficient low-rank approximation algorithms.

5.4 Weak error estimates for rough volatility models

Thomas Wagenhofer, TU Wien

We consider a rough volatility model where the volatility is a (smooth) function of a Riemann-Liouville Brownian motion with Hurst parameter H in $(0, 1/2)$. When simulating these models, one often uses a discretization of stochastic integrals as an approximation. These integrals can be interpreted as log-stock-prices. In Applications, such as in pricing, the most relevant quantities are expectations of (payoff) functions.

Our main result is that moments of these integrals have a weak error rate of order $3H+1/2$ if $H < 1/6$ and order 1 otherwise. For this we first derive a moment formula for both the discretization and the true stochastic integral. We then use this formula and properties of Gaussian random variables to prove our main theorem. We furthermore provide a lower bound, showing the optimality of that rate. Note that our rate of $3H+1/2$ is in stark contrast to the strong error rate which is of order H .

This is a joint work with Peter Friz and William Salkeld

5.5 Polynomial Volterra processes

Luca Pelizzari, Weierstrass Institute

We study a novel class of stochastic Volterra processes, the so-called polynomial Volterra processes, which are solutions to stochastic Volterra equations, where the characteristics of the driving semimartingale have polynomial structure. We provide a full analysis of existence and uniqueness for the new class of processes and prove a characterization of the moments as solutions to systems of certain Volterra-type integral equations. We build a new rough stochastic volatility model driven by polynomial Volterra processes, the rough Jacobi model, which generalizes the known Jacobi stochastic volatility model. We prove that this new model contains the rough Heston model as a limit case, and we derive numerical methods to solve the Volterra-type integral equations for the moments.



6. Further Topics in Stochastic Analysis

6.1 Quenched functional CLT for random walks in degenerate doubly stochastic random environments

Benjamin Walker, University of Oxford

Neural controlled differential equations (NCDEs) are a state-of-the-art method for modelling irregularly sampled multivariate time series data. Current methods for training a NCDE require rewriting the CDE as an ordinary differential equation (ODE). This is done by either taking the control to be a *differentiable* interpolation of the data or by using the Log-ODE method. The Log-ODE method has proved to be a powerful approach when handling long time-series.

Currently, the Log-ODE's vector field is modelled as an arbitrary neural network. However, the Log-ODE vector field is a linear combination of the iterated Lie brackets of the CDE's vector field. This work explores how this structure can be employed to increase the performance of NCDEs.

6.2 Quantitative John–Nirenberg inequality for stochastic processes of bounded mean oscillation

Khoa Lê, University of Leeds

Stroock and Varadhan in 1997 and Geiss in 2005 independently introduced stochastic processes with bounded mean oscillation (BMO) and established their exponential integrability with some unspecified exponential constant. This result is an analogue of the John–Nirenberg inequality for functions of bounded mean oscillation. In this work, we quantify the size of the exponential constant by the modulus of mean oscillation. Some new applications of BMO processes in rough stochastic differential equations, numerical approximations and regularization by noise are discussed.

6.3 A Forward-Backward SDE Approach to the Sine-Gordon QFT

Sarah-Jean Meyer, University of Oxford

We present a construction of the massive sine-Gordon quantum field theory (QFT) on the infinite volume \mathbb{R}^2 using a stochastic quantisation via a forward-backward stochastic differential equation (FBSDE). A stochastic analysis of the FBSDE allows us to construct the measure on the full space for $2 < \beta < 4$. For a large mass or weak coupling, this measure is shown to be unique. Our approach builds on the variational method for Euclidean QFTs introduced by BarashkovGubinelli and relies on the (martingale) structure of the continuous renormalisation group.

6.4 Quenched functional CLT for random walks in degenerate doubly stochastic random environments

Weile Weng, TU Berlin

Invariance principles have been extensively studied for random conductance model, in which the speed of a particle that performs random walks is symmetric on edges. What about the random environments that allow non-symmetric speed? We consider doubly stochastic random environments on \mathbb{Z}^d ($d \geq 2$) that are stationary and ergodic in space. We investigate the quenched functional CLT (a.k.a. quenched invariance principle) for continuous-time variable speed random walks in such random environments that sometimes can be degenerate. Precisely, instead of the usually imposed uniform ellipticity and boundedness conditions on the speed, we assume moment conditions on the its symmetric part, and on the stream cycles that form the anti-symmetric part. In this talk, I will introduce the model and discuss the main ingredients of the proof.

6.5 Analysis of the Ensemble Kalman–Bucy Filter for correlated observation noise

Sebastian Ertel, TU Berlin

The Ensemble Kalman–Bucy filter (EnKBF) is an important tool in the field of stochastic filtering, that aims to approximate the law of a diffusion process, called the signal, conditioned on noisy observations. This is achieved by employing a system of diffusion processes interacting through their ensemble mean and covariance. In this talk we first derive an EnKBF applicable to the correlated noise framework, that is when the evolution of the signal and the observation process are both influenced by a common noise term. We prove the well-posedness of the EnKBF, which in the correlated case is a singular stochastic differential equation and requires controlling the (pseudo)inverse of the ensemble covariance matrix. Finally we investigate the mean-field limit, which is given by a singular McKean–Vlasov equation, that only satisfies local Lipschitz conditions. We prove the well-posedness of the equation and a propagation of chaos result.

6.6 Dobrushin and Steif Metrics are Equal

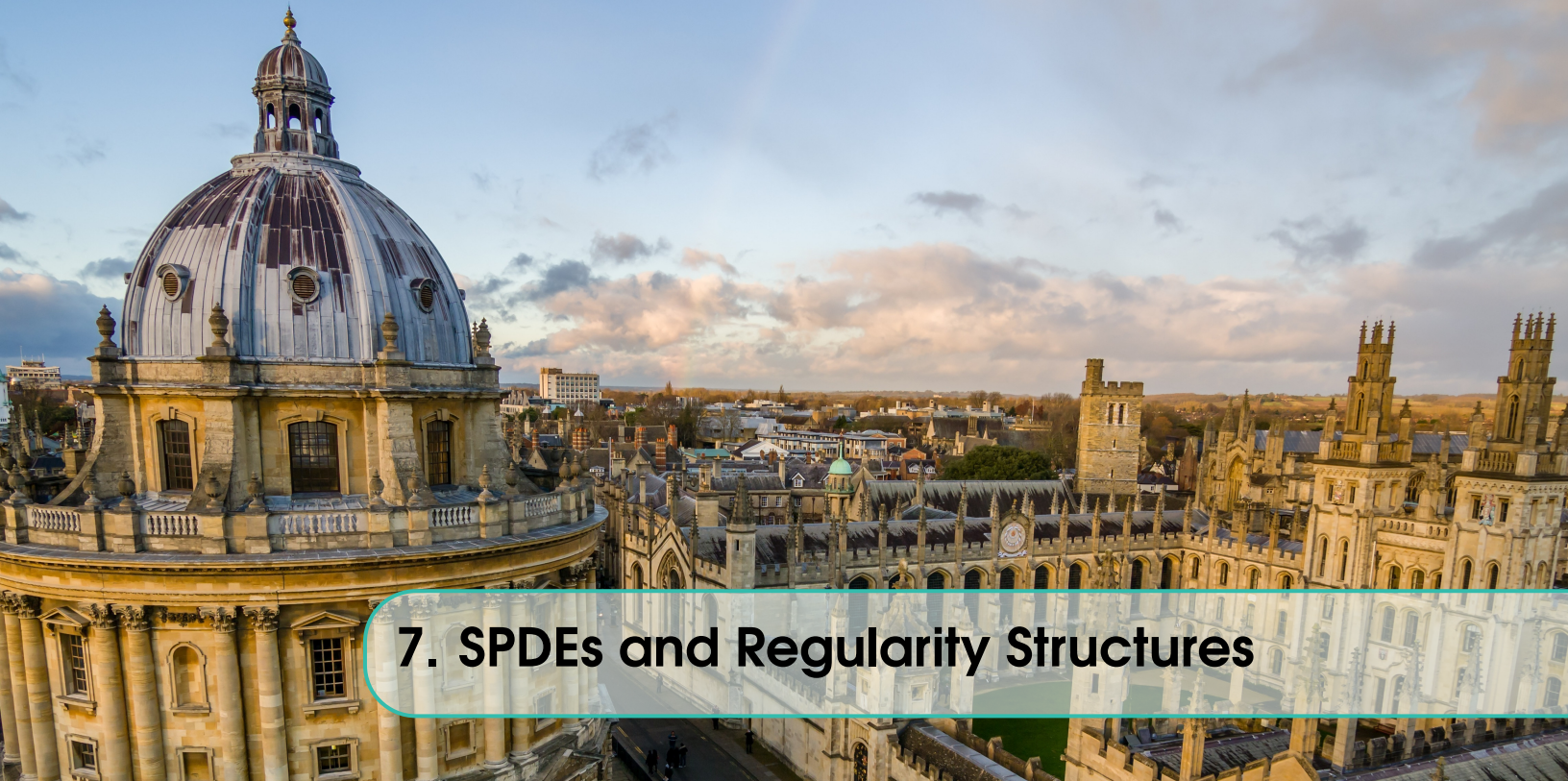
Jacob Armstrong Goodall, University of Edinburgh

It is well-known that standard metrics on spaces of multivariate probability distributions with many or countably infinite number of variables are of limited use. For example, Liggett laments on p.70 of 'Interacting particle systems' that "total variation convergence essentially never occurs for particle systems". To aid this, in 2011 Robert MacKay introduced a metric on multivariate probability distributions that does give convergence for many systems. It turned out that Jeffery Steif had introduced a metric achieving the same goal in 1991. In 2020 I proved that that the two metrics are in fact equal, paving the way for advances in their computation. This talk will be about that proof.

6.7 Large deviations of slow-fast systems driven by fractional Brownian motion

Ioannis Gasteratos, Imperial College London

We consider a multiscale system of stochastic differential equations in which the slow component is perturbed by a small fractional Brownian motion with Hurst index $H > 1/2$ and the fast component is driven by an independent Brownian motion. Working in the framework of Young integration, we use tools from fractional calculus and weak convergence arguments to establish a Large Deviation Principle in the homogenized limit, as the noise intensity and time-scale separation parameters vanish at an appropriate rate. Our approach is based in the study of the limiting behavior of an associated controlled system. We show that, in certain cases, the non-local rate function admits an explicit non-variational form. The latter allows us to draw comparisons to the case $H = 1/2$ which corresponds to the classical Freidlin-Wentzell theory. Moreover, we study the asymptotics of the rate function as $H \rightarrow 1/2^+$ and show that it is discontinuous at $H = 1/2$. This is a joint work with Siragan Gailus.



7. SPDEs and Regularity Structures

7.1 SPDE Approximations to Singular Mean-Field Systems

Adrian Martini, University of Oxford

To achieve the formation of complex structures, many biological populations employ a mechanism called chemotaxis. In this process, individuals disperse a chemical to which other members can react. One can model such a population with a mean-field system of particles interacting through a Coulomb force. However, owing to the singularity of the interaction, this model is mathematically involved. On the other hand, we can also interpret the same particle system as a formal solution to a stochastic PDE that consists of the mean-field limit plus a non-linear noise capturing intrinsic, finite-number fluctuations. In this talk, we introduce an SPDE with additive noise to approximate finite, chemotactic populations. We establish the well-posedness of this equation through the theory of paracontrolled distributions and discuss its renormalisation. We can then quantify the accuracy of our approximation through a large deviation principle.

7.2 Singular Equations on homogeneous Lie Groups

Harprit Singh, Imperial College London

We show that Regularity Structures, without much change, work on general homogeneous Lie groups. In particular we obtain a solution theory for SPDEs where the differential operator stems from a large class of hypo-elliptic operators. Joint work with A. Mayorcas.

7.3 The BPHZ Theorem for Models via Spectral Gap

Rhys Steele, Imperial College London

In this talk, I will discuss a new proof of the convergence of the BPHZ renormalised models in the setting of regularity structures for semilinear singular SPDEs. This work is inspired by recent work of Linares, Otto, Tempelmayr and Tsatsoulis in the quasilinear setting and thus takes a spectral gap assumption on the driving noise as its starting point.

Our approach relies crucially on a novel version of the reconstruction theorem for a space of “pointed Besov modelled distributions”. As a consequence, the analytical core of the proof is quite short and self-contained; in contrast to the original proof of Chandra and Hairer.

(Based on a joint work with Martin Hairer)

7.4 Unique $L^2(\mu)$ -semigroups for some (sub-)critical singular SPDEs

Lukas Gräfner, FU Berlin

Continuing the method from [GP20], we present a probabilistic approach for some singular SPDEs with known (in principle not necessarily invariant or Gaussian) reference measure. Our central tool is a perturbation result for semigroups on abstract Fock spaces which yields a unique $L^2(\mu)$ -semigroup, describing a unique Markov process, that is the solution to the underlying equation. Applications include some singular, scaling-critical equations. This is joint work with Nicolas Perkowski.

[GP20] Gubinelli, M., Perkowski, N. The infinitesimal generator of the stochastic Burgers equation. *Probab. Theory Relat. Fields* 178, 1067–1124 (2020).

7.5 Novikov algebra structure for multi-indices, aromatic B-series and numerical methods

Jean-David Jacques, Sorbonne Université

Recent work on regularity structures has led to a structure of Novikov algebra on the free vector space generated by multi-indices, which has been used in the past for the study of universal geometric properties of aromatic B-series. After giving a brief introduction about numerical methods leading to aromatic B-series, I will talk about my recent work on pre-Lie structures for multi-indices and its associated Hopf algebra, which is an analogue of the dual of the Connes-Kreimer algebra.



8. Participants

Martin Geller (University of Oxford)
Nikkita Ngalande (University of Oxford)
Henry Chiu (Imperial College London)
Harprit Singh (Imperial College London)
Xianfeng Ren (Imperial College London)
Milena Vuletić (University of Oxford)
Yifan Jiang (University of Oxford)
Sarah-Jean Meyer (University of Oxford)
Rhys Steele (Imperial College London)
Yuhang He (University of Oxford)
Tom Morrish (University of Warwick)
Satoshi Hayakawa (University of Oxford)
Lingyi Yang (University of Oxford)
Darrick Lee (University of Oxford)
Jonathan Tam (University of Oxford)
Karen Habermann (University of Warwick)
Yihuang Zhang (University of Oxford)
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