Rough Paths

DataSig

A rough path between mathematics and data science

Terry Lyons

26 February 2021

with many others… but particularly Tom, Varun, Cris, Maud, Peter, Roly, Sam, Patricia

University of Oxford · The Alan Turing Institute · Imperial College London · UCL · UKRI Engineering and Physical Sciences Research Council

A Turing vision

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> We channel our research around a number of ambitious challenges which represent areas in which AI and data science can have a game-changing impact for science, society, and the economy. These challenges will not be led by the Turing alone, but depend on significant collaboration and partnerships.

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Modelling behavior of evolving systems

A collaboration

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GCHQ

Thomas C

Jack D

National Cyber Security Centre
a part of GCHQ

David P

dstl

Varun C

DataSig

Terry Lyons

Cris Salvi

Patricia Andrew

Imanol Pérez Arribas

The Alan Turing Institute

Peter Foster

Maud Lemercier

Roly Perera

DataSig | an EPSRC/UKRI 5-year program grant

Mathematics

- rough path theory and signatures
- describing the interactions between complex systems from the top down
- extending the calculus of differential equations to complex contexts

Data science

- the notion of an unparameterised path captured by the order of events
- clean and minimal universal feature sets
- the notion of a neural controlled differential equation
- The notion of a pde-kernel
- a principled mathematical framework that allows further innovation

Embedded contexts

- streamed data is everywhere; Chinese handwriting, hospital wards, …

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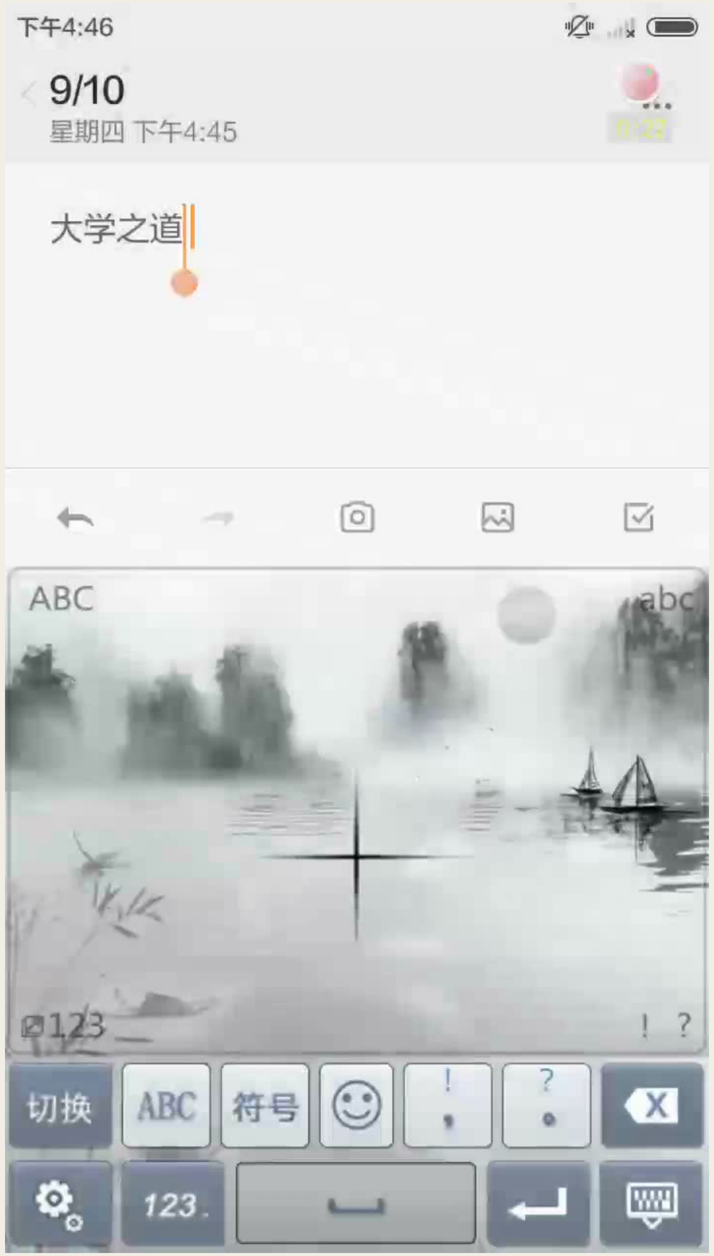

Streamed data

- a character drawn on the screen of an iPhone
- an order book
- a piece of text
- progression through hospital record
- astronomical data
- video of a person moving
- an evolving stream of emotions
- ICU data to detect sepsis

Ensembles of streamed data

- the processes generated by malware
- the behaviour of crowds
- the evolution of cancer cell lines

Key questions

- understand what you have observed
- predict the distribution of what is happening next
- identify anomalies

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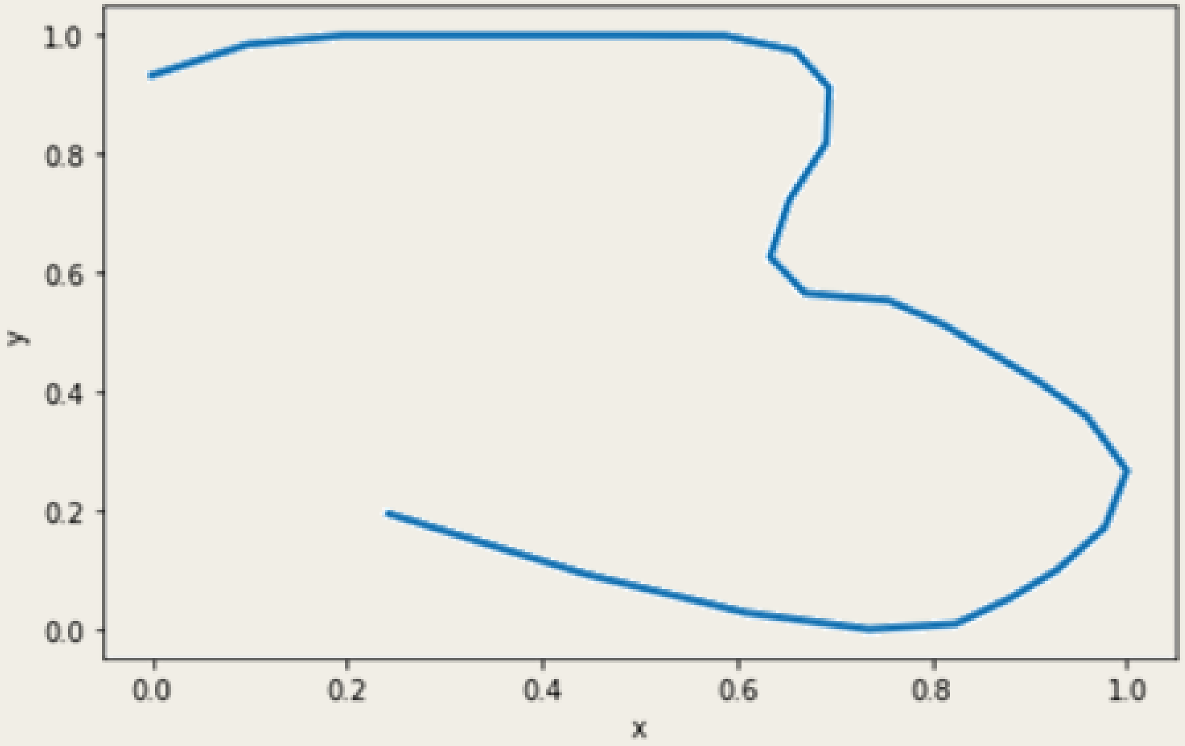
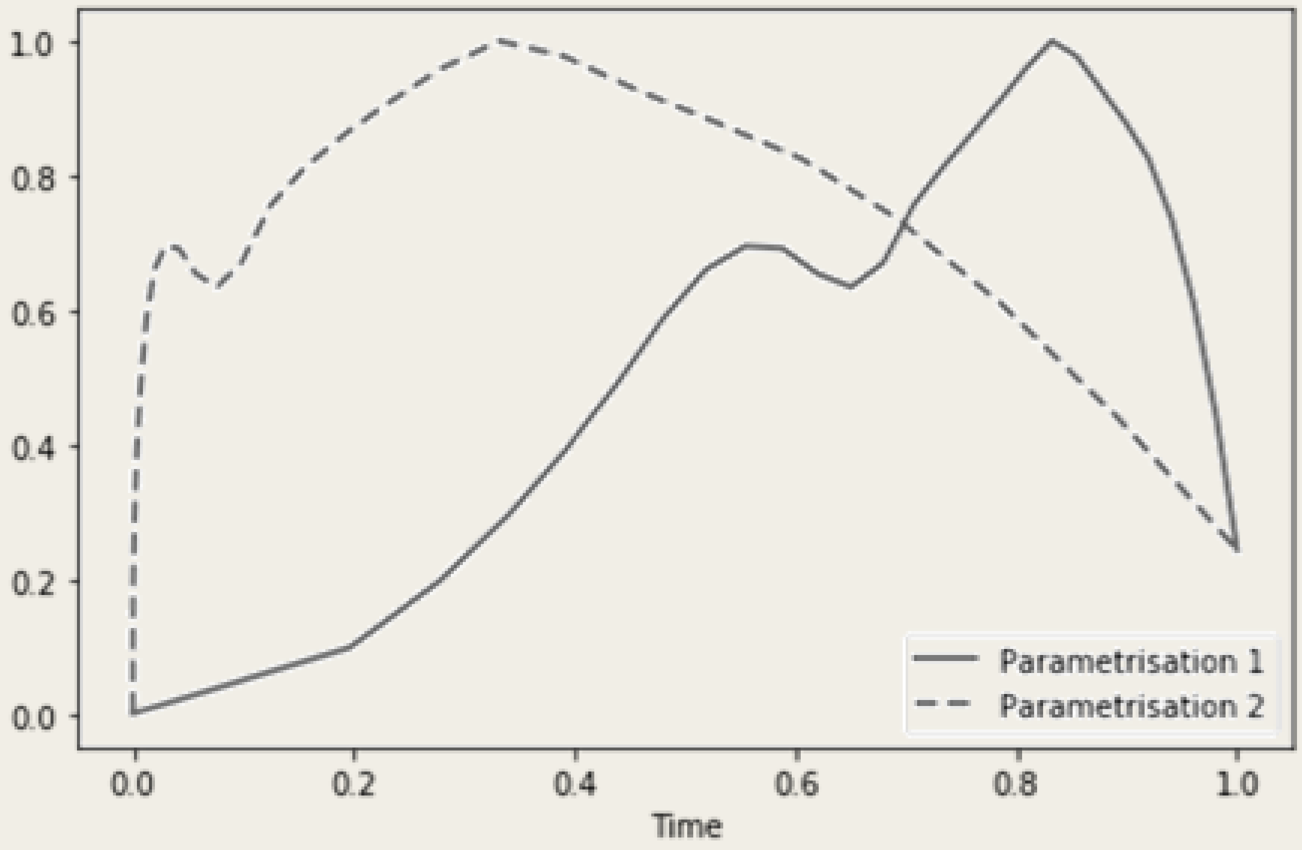
Some maths of evolving systems

Data science does not like symmetry

- Re-parameterisation is a huge symmetry group
- Multimodal streams modulo re-parameterisation form a group
- Representing this group in the tensor algebra provides a faithful feature set and removes the symmetry
- New tools signature and log signature, new maths describing the functions on streams



Different sampling procedures

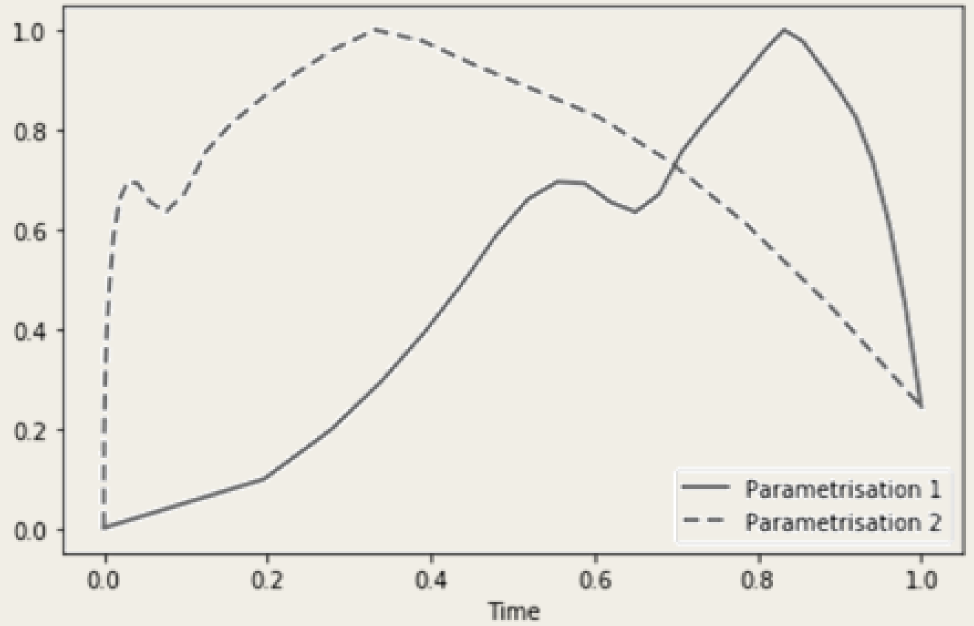


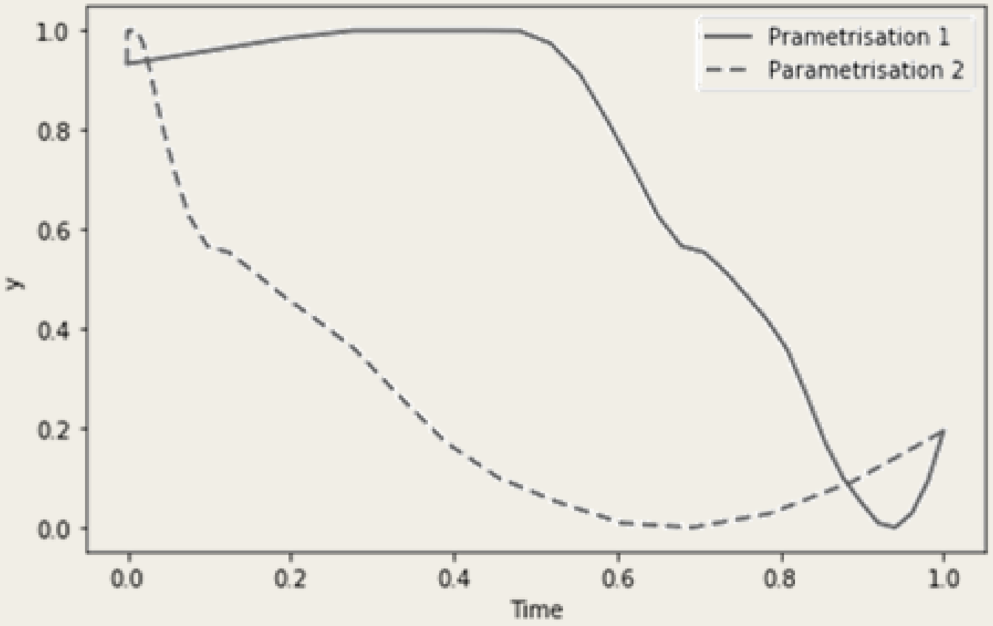
- The letter “3” is drawn from top to bottom
- The x coordinate of the evolving symbol sampled differently (at uneven speeds)

Different sampling procedures

The number “3” x, y coordinates – same picture drawn at two different speeds

- no consistent wavelets
- reparameterisations do not form a linear space!





Different sampling procedures

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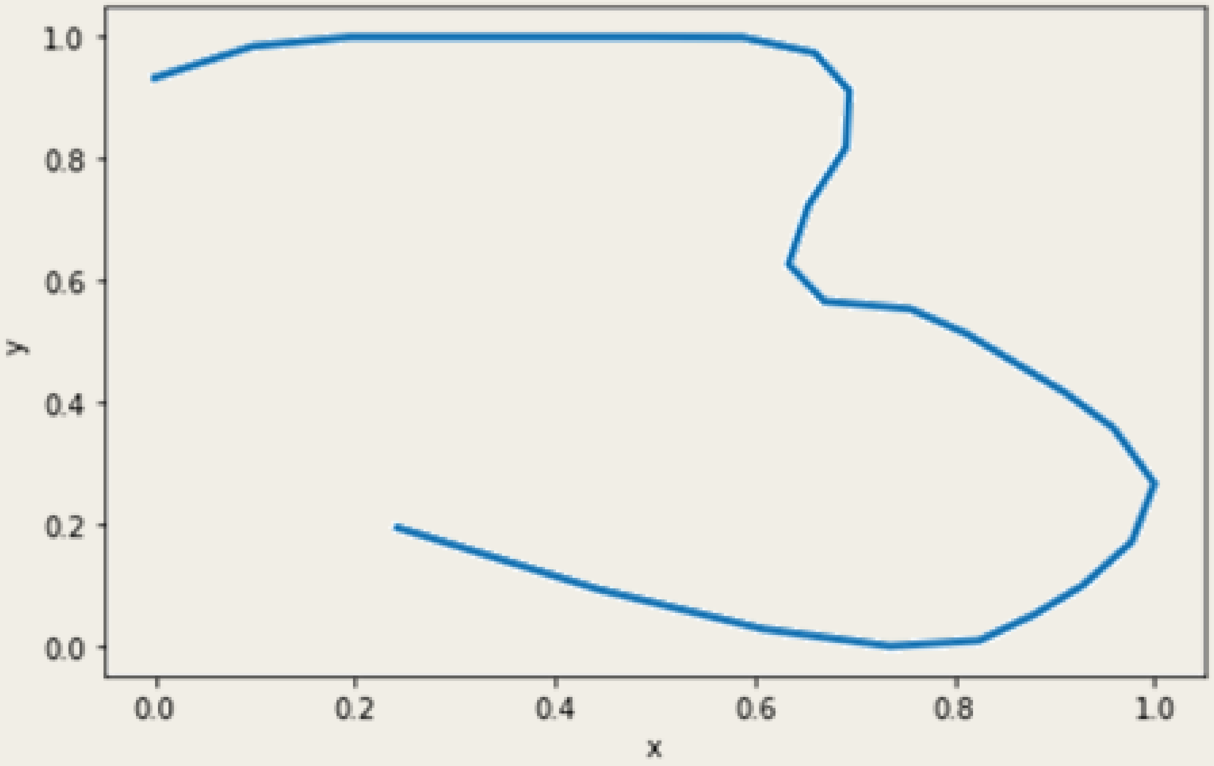
- The letter “3” is drawn from top to bottom
- How does one describe the three or any path modulo the symmetry of parametrisation?



The signature of a path describes an unparameterised stream γ

Signature is a *top down* description for unparameterised paths that describes a path segment through its effects of stylised nonlinear systems

$$dS = S \otimes d\gamma$$

It filters out the infinite dimensional noise of resampling allowing prediction and classification with *much* smaller learning sets.

It gives fixed dimensional feature sets regardless of the sample points.*

* missing data/varying parameterisation not issues although inadequacy may be

Analysis, geometry, combinatorial Hopf/dendriform/sensor algebras

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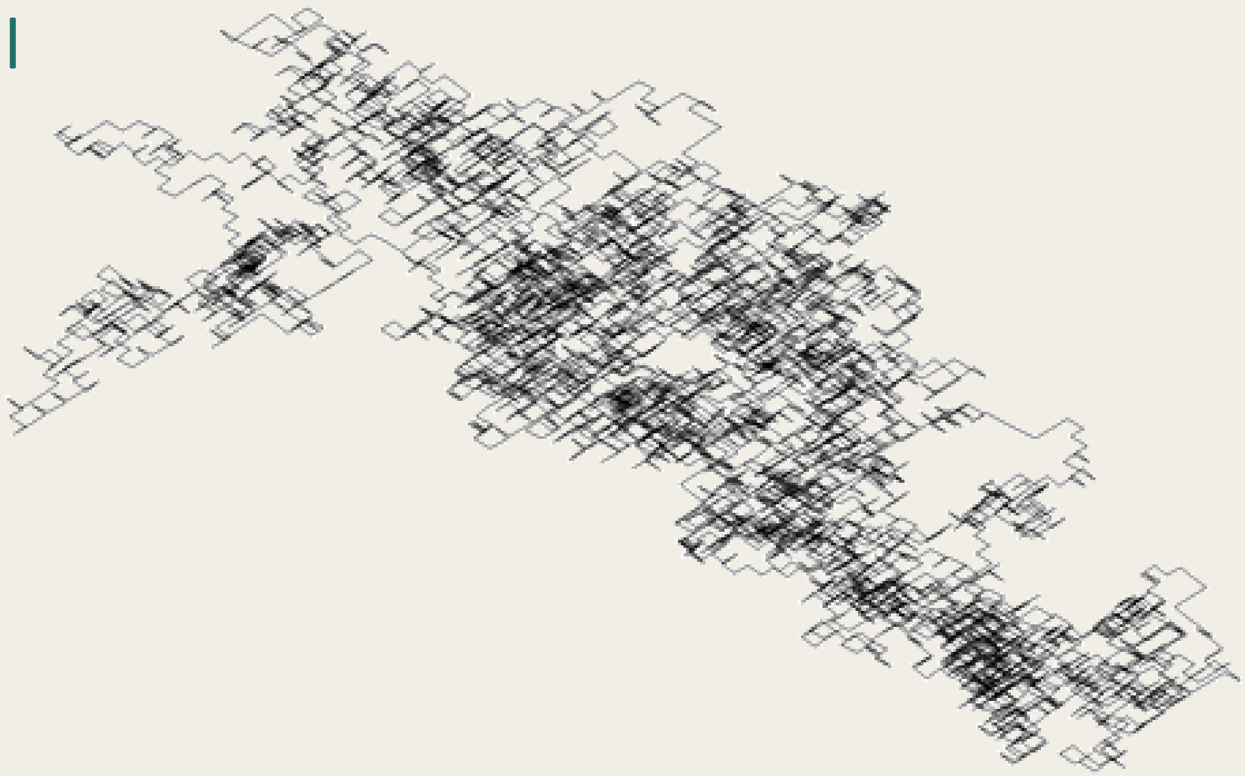
Signature leads to linear space of real valued functionals $\langle \boldsymbol{e} | \boldsymbol{S_I} \rangle$ on streams

Pointwise multiplication and integration of these functionals

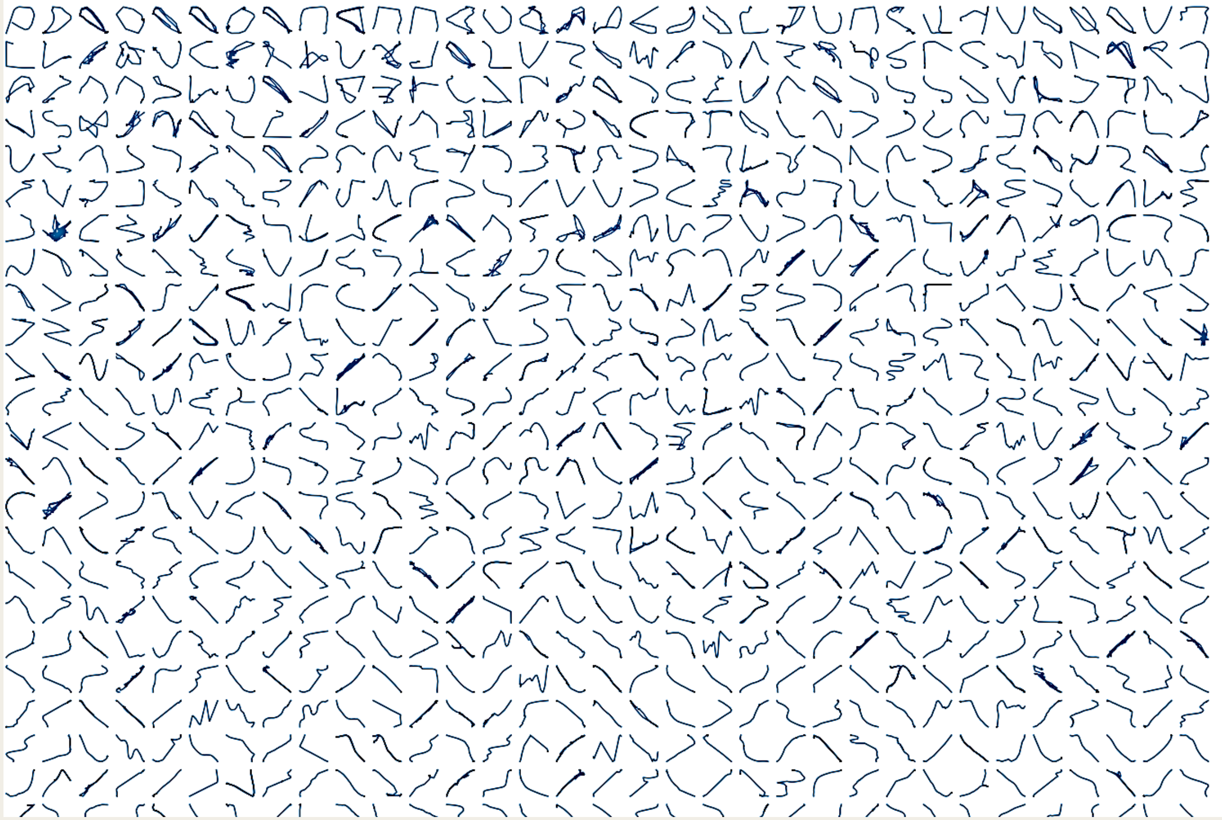
$$\langle \boldsymbol{\alpha} | \boldsymbol{\gamma} \rangle \langle \boldsymbol{\beta} | \boldsymbol{\gamma} \rangle = \langle \boldsymbol{\alpha} \Cup \boldsymbol{\beta} | \boldsymbol{\gamma} \rangle$$

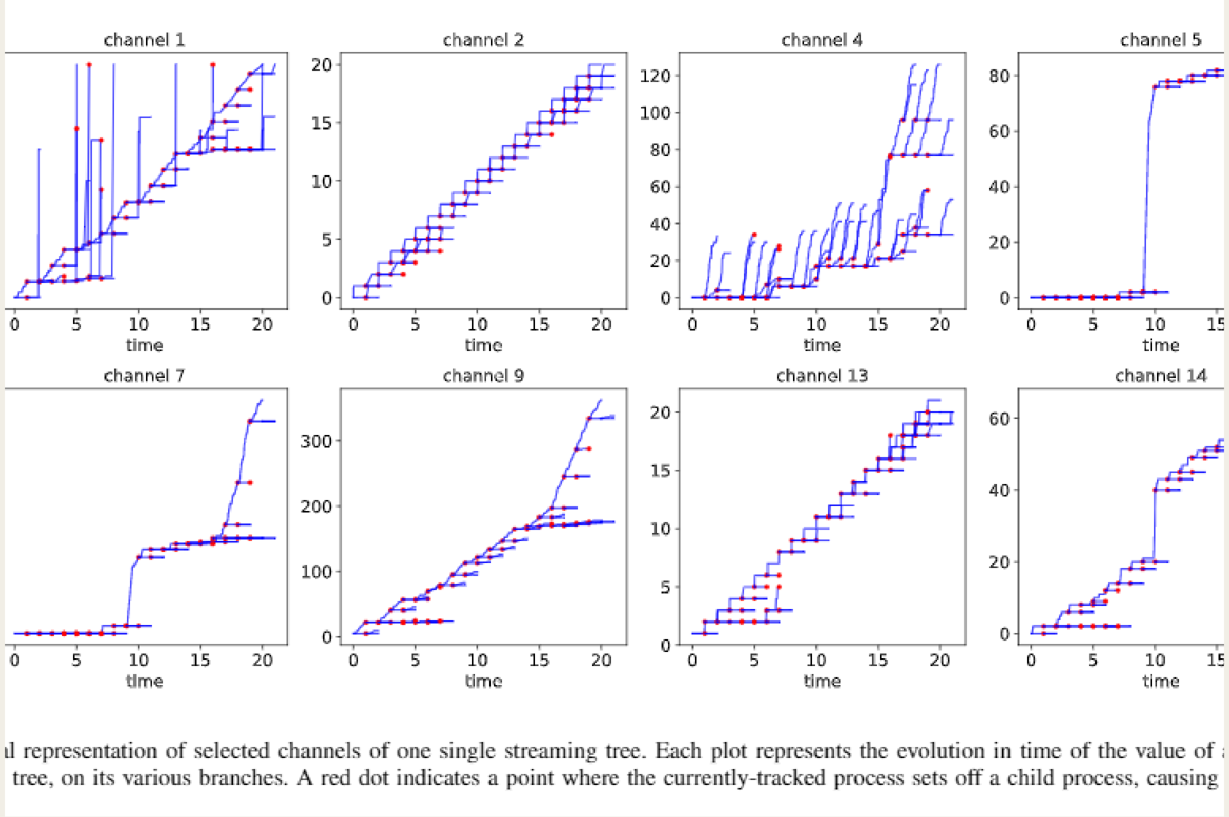
$$\int \langle \boldsymbol{\alpha} | \boldsymbol{\gamma} \rangle \, \boldsymbol{d} \langle \boldsymbol{\beta} | \boldsymbol{\gamma} \rangle = \langle \boldsymbol{\alpha} \prec \boldsymbol{\beta} | \boldsymbol{\gamma} \rangle$$

can usefully be described in purely algebraic language.



Our data




channel 1
channel 2
channel 4
channel 5
channel 7
channel 9
channel 13
channel 14
time
l representation of selected channels of one single streaming tree. Each plot represents the evolution in time of the value of
tree, on its various branches. A red dot indicates a point where the currently-tracked process sets off a child process, causing

Recovering the curves from the signature

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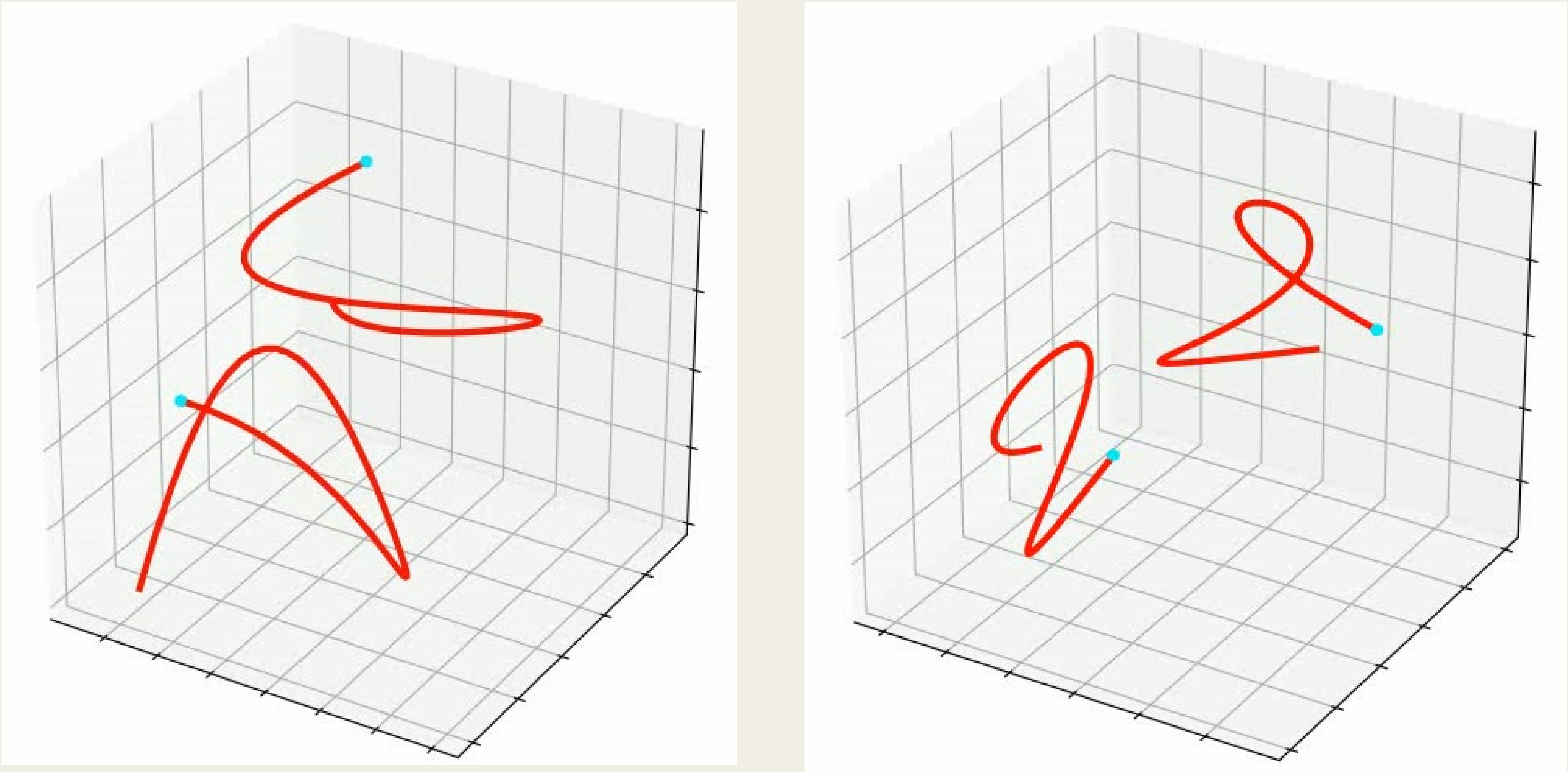

Vectorisation of unsampled streams

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- signatures connect streams to the data science pipeline, but allows graduated approach
- expected signatures describe ensembles of paths
- pdes provide kernels on paths

Our collaboration

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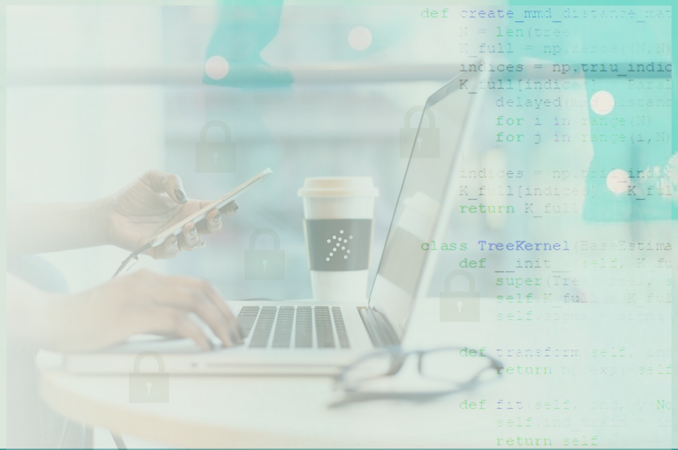
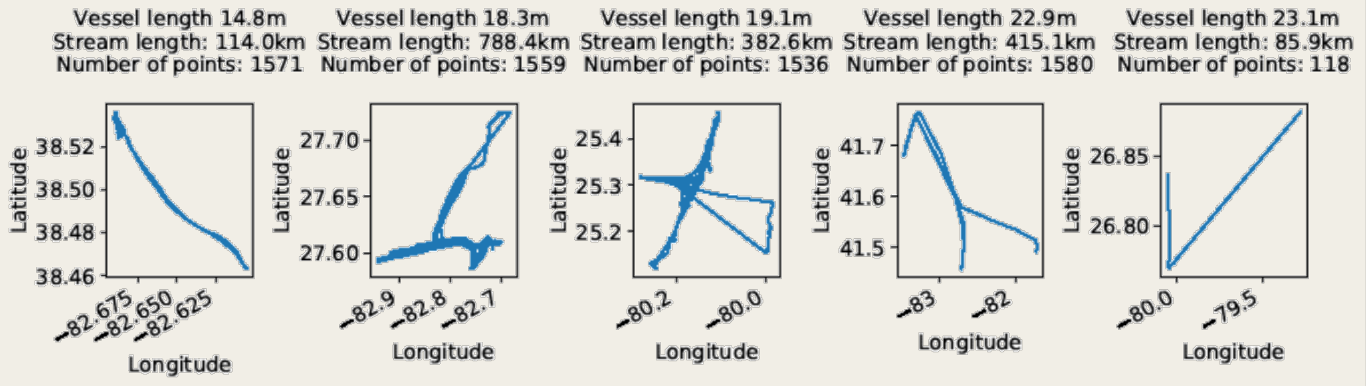
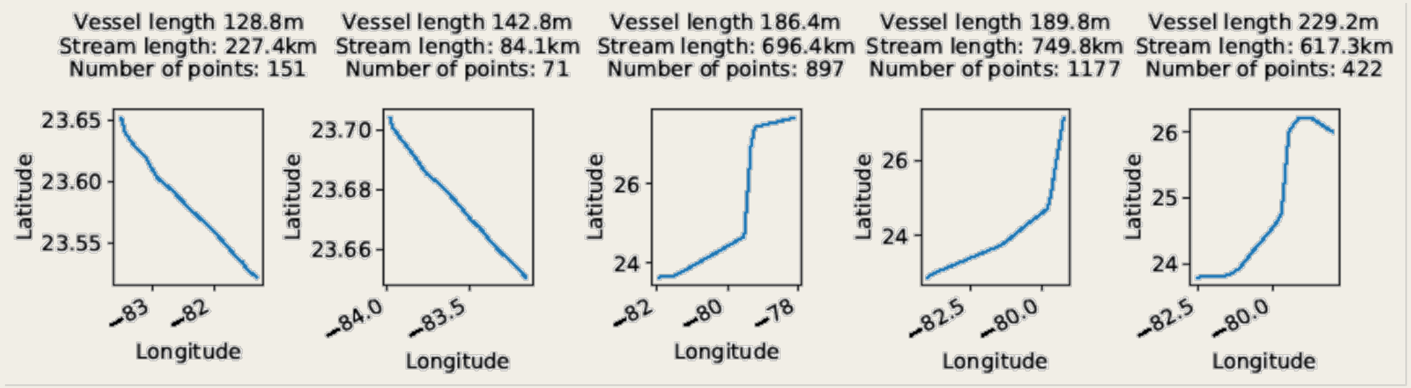


Signatures or Paths? |Long / Short vessel

- Demonstrated that novel math based method for anomaly detection is widely applicable and can detect anomalous streams where other methods fail.
- Applied this to the real-world shipping trajectories example.





Process tree example : Expected signatures of clouds of paths

Developed a way to apply expected signature techniques to host-based event logs, by viewing processes as trees evolving over time.

Applied this to malware classification: early results are that signature features improve classification of malware; further investigations are ongoing on richer data sets.

Process tree example : Expected signatures of clouds of paths

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Developed SK-tree structure to apply standardised expected signature techniques to host-based event logs, by viewing processes as trees evolving over time analysed as expected signatures through a PDE kernel.

2102.07904.pdf (arxiv.org)

We demonstrate the SK-Tree to detect malicious events on a portion of the publicly available DARPA OpTC dataset, achieving an AUROC score of 98%

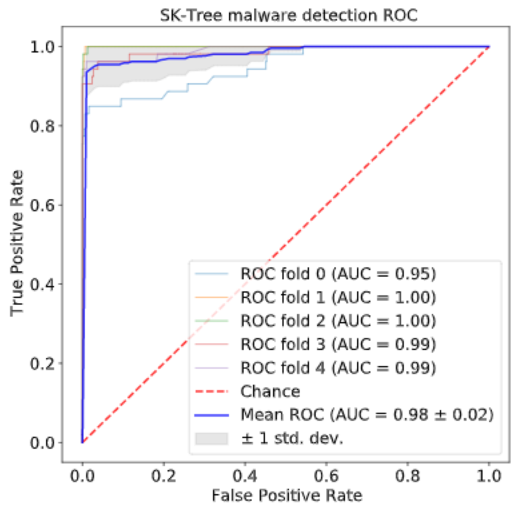
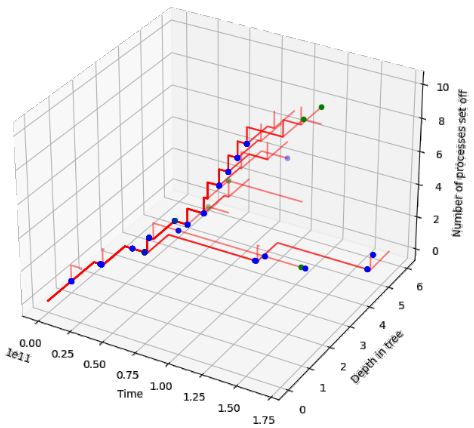
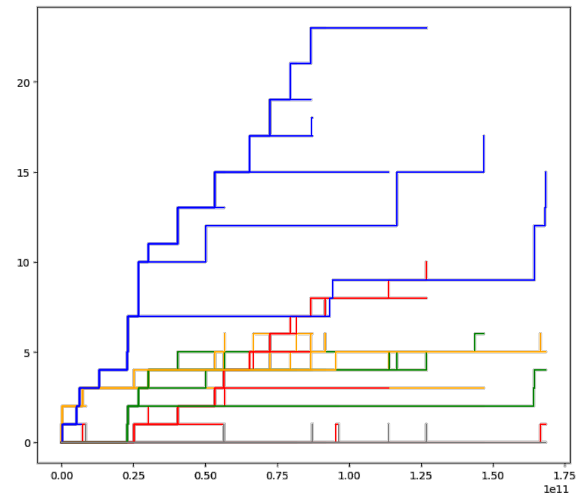


Fig. 2. ROC evaluation of the SK-Tree binary classifier on the OpTC data





Landmark based action recognition

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To communicate our methodology, we construct notebooks with introductory examples of what we can do.

People moving can easily be anonymized to landmarks. It is a static process. The moving stick people still contain information.

Peter Foster has put together a simple notebook you can run that demonstrates viable approaches to recognizing these actions that can be trained on small datasets.

Thanks for listening and over to Peter!

https://www.datasig.ac.uk/people